Electromagnetic waves

- Waves, mean of transporting energy or information
- Wave is a function of both space and time
- w → angular frequency
- f → frequency (number of cycles per second)
- λ → wave length

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1. Faraday's Law

magnetic form: $-V_{emf} = \frac{d\phi}{dt}$ ϕ → flux through each turn

electric form: $-V_{emf} = \oint E \cdot dl = IR$

* Maxwel Said

$$\therefore \oint E \cdot dl = \frac{d\phi}{dt} \quad \rightarrow (1)$$

where $\phi = \oint B \cdot dS$

$$\frac{d}{dt}(\phi) = \frac{d}{dt} \oint B \cdot dS \quad \rightarrow (2)$$

From (1) & (2)

$$\oint E \cdot dl = -\frac{d}{dt} \oint B \cdot dS$$

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→ From Stakes's theorem

$$\int_S (\nabla \times \bar{E}) \cdot dS = \oint E \cdot dL$$

$$\therefore \int_S (\nabla \times \bar{E}) \cdot dS = -\frac{d}{dt}\int_S B \cdot dS$$

$$\therefore \nabla \times \bar{E} = \ominus \frac{dB}{dt}$$

From lenz's Law, the induced current I flows to produce a magnetic field that opposes (B).

$$\nabla \times \bar{E} = -\frac{\partial B}{\partial t}$$ 1st eqn of Maxwell's point form.

if B is constant $\frac{\partial B}{\partial t} = 0$

$$\therefore \nabla \times \bar{E} = 0$$

So we study Time varying field.

this eqn show that a time changing magnetic field produce an electric field.

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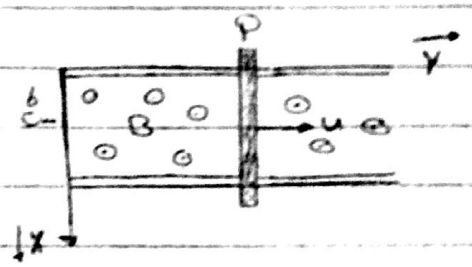
Example :

$B = 4 \cos 10^6 t \; a_z \; wb/m^2$

$y = 8 \; cm$



Find V_{emf} ?

$$V_{emf} = -\int \frac{\partial B}{\partial t} \cdot dS$$

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$$\frac{\partial B}{\partial t} = -4 * 10^6 \sin 10^6 t$$

$$\therefore V_{emf} = \int 4*10^6 \sin 10^6 t \; . dx \, . dy$$

$$= \int_{x=0}^{6\,cm} \int_{y=0}^{8\,cm} 4 * 10^6 \sin 10^6 t \; dx \; dy$$

$$= .06 * .08 * 4*10^6 \sin 10^6 t$$

$$= 19200 \sin 10^6 t \quad volt$$

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Ampere's Law

$$I_{enc} = \oint H \cdot dL$$

and $$\nabla \times \bar{H} = J$$

$$\therefore \nabla \cdot (\nabla \times H) = \nabla \cdot J = 0 \quad \text{(non equal zero)}$$

But $$\nabla \cdot J = \frac{\partial \rho_v}{\partial t}$$

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Displacement Current

$$\nabla \times \bar{H} = J + G$$

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot J + \nabla \cdot G$$

$$\downarrow$$

$$0 = \nabla \cdot J + \nabla \cdot G$$

$$0 = -\frac{\partial \rho_v}{\partial t} + \nabla \cdot G$$

$$\therefore \nabla \cdot G = \frac{\partial \rho_v}{\partial t}$$

where $$\rho_v = \nabla \cdot D$$

$$\therefore \nabla \cdot G = \frac{\partial}{\partial t} \nabla \cdot D = \nabla \frac{\partial D}{\partial t}$$

$$\therefore G = \frac{\partial D}{\partial t}$$

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$$\nabla \times \bar{H} = J + \frac{\partial D}{\partial t}$$

$$= J_c + J_d$$

J_c : Conduction Current density ($J_c = \sigma E$)

J_d : Displacement Current density ($J_d = \frac{\partial D}{\partial t}$) $D = \varepsilon E$

* Maxwell's equation in point form

① $\nabla \times \bar{E} = -\frac{\partial B}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t}$

② $\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t} = \sigma \bar{E} + \frac{\partial \varepsilon \bar{E}}{\partial t}$

③ $\nabla . D = \rho_v$

④ $\nabla . B = 0$

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* Maxwell's equation in integral form

① $\oint \bar{E} . d\bar{L} = -\int_s \frac{\partial B}{\partial t} . dS$

② $\oint \bar{H} . d\bar{L} = I + \int_s \frac{\partial D}{\partial t} . dS$

③ $\oint_s D . dS = \int_{vol} \rho_v \, dv = Q_{enc.}$

④ $\oint_s B . dS = 0$

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- Wave propagate in a general medium (lossy medium)

(E & H has only one component { E_x or E_y or E_z } & { H_x or H_y or H_z })

assume

$$\bar{E} = \bar{E}_x \hat{a}_x$$

$$\bar{H} = \bar{H}_y \hat{a}_y$$

- From maxwell's equations

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$$\textcircled{1} \quad \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\begin{vmatrix} +\hat{a}_x & -\hat{a}_y & +\hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{E}_x \left(\frac{\partial}{\partial z} \hat{a}_y - \frac{\partial}{\partial y} \hat{a}_z \right) = -\frac{\partial \bar{B}}{\partial t} \qquad B = \mu \bar{H} \hat{a}_y$$

no comp. at direct z for $\bar{H}$

$$\frac{\partial \bar{E}_x}{\partial z} \hat{a}_y = -\mu \frac{\partial H}{\partial t} \longrightarrow \textcircled{1}$$

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From equation ① E must have Component of (z)

$$\frac{\partial E}{\partial z} = \neq$$

if E has no z comp.

$$\frac{\partial E}{\partial z} = \text{Zero}.$$

From the Maxwell's equation

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② $\nabla \times \bar{H} = \bar{J}_c + \bar{J}_{cl}$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \bar{H}_x & \bar{H}_y & \bar{H}_z \end{vmatrix} = \sigma \bar{E} + \frac{\partial \bar{D}}{\partial t}$$

$$-H_y \left(\frac{\partial}{\partial z} \hat{a}_y - \frac{\partial}{\partial x} \hat{a}_z \right) = \sigma E_x + \varepsilon \frac{\partial E_x}{\partial t}$$

$$\frac{\partial H_y}{\partial z} \hat{a}_x + \cancel{\frac{\partial H_y}{\partial x}}^{0} \hat{a}_z = \sigma E_x + \varepsilon \frac{\partial E_x}{\partial t}$$

$$\therefore \frac{\partial \bar{H}_y}{\partial z} \hat{a}_x = -E_x \left(\sigma + \varepsilon \frac{\partial}{\partial t} \right) \quad (2)$$

also H must have z Component

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* وهوا يا معنى الـ Plane wave

① عندي مركبة الـ E في اتجاه واحد فقط ✓ i.e. x ✓

② و ، ، ، H في اتجاه عمودي ✓ i.e y ✓

③ هيبقى اتجاه الانتشار الـ power بتاعهم عمودي عليهم وفي اتجاه الـ (Z) .

$\longrightarrow$ نشر

الآن نهدف نحن

* now we aim to get form represent E propagation and H propagation

الآن نهدف إلى الحصول على الشكل

From ①

$$\nabla \times \bar{E} = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times \nabla \times \bar{E} = -\mu \frac{\partial}{\partial t} (\nabla \times H)$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = -\mu \frac{\partial}{\partial t} \left(\sigma E + \varepsilon \frac{\partial E}{\partial t} \right)$$

$\nabla(\nabla \cdot E) \rightarrow 0$

$$-\nabla^2 E = -\mu \frac{\partial}{\partial t} \left(\sigma E + \varepsilon \frac{\partial E}{\partial t} \right)$$

$$\frac{\partial}{\partial t} = j\omega$$

$$\nabla^2 E = \mu (j\omega) (\sigma E + \varepsilon j\omega E)$$

$$= j\omega\mu\sigma E - \omega^2 \mu \varepsilon E$$

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$$\nabla^2 E = (j\omega\mu\sigma - \omega^2\mu\varepsilon)\, E$$

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 E - \gamma^2 E = 0$$

$$E_x = E_x^+ e^{-\gamma z} + E_x^- e^{\gamma z}$$

$$\therefore \bar{E} = E^+ e^{-\gamma z}\, \hat{a}_E + E^- e^{\gamma z}\, \hat{a}_E$$

also

$$\bar{H} = H^+ e^{-\gamma z}\, \hat{a}_H + H^- e^{\gamma z}\, \hat{a}_H$$

$$\because \gamma^2 = j\omega\mu\sigma - \omega^2\mu\varepsilon$$

$$\gamma = \sqrt{(j\omega\mu\sigma - \omega^2\mu\varepsilon)}$$

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$$\gamma = (j\omega\mu\sigma - \omega^2\mu\varepsilon)^{1/2}$$

propagation Constant

$$\gamma = \alpha + j\beta$$

attenuation Constant — تحقيق

phase Constant , wave number

- Calculate (α & β)

$$\gamma^2 = \alpha^2 - \beta^2 + 2j\alpha\beta$$

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$$\therefore \quad Re(\gamma^2) = -\omega^2 \mu \varepsilon$$

$$\therefore \quad \alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon$$

and

$$|\gamma^2| = \alpha^2 + \beta^2 = \sqrt{(\omega\mu\sigma)^2 + (\omega^2\mu\varepsilon)^2}$$

$$\alpha^2 + \beta^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2}$$

$$\therefore \quad 2\alpha^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2} - \omega^2\mu\varepsilon$$

$$2\alpha^2 = \omega\left(\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2} - \omega\mu\varepsilon\right)$$

$$2\alpha^2 = \omega^2\mu\varepsilon\left[\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1} - 1\right]$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1} - 1\right]} \rightarrow (1)$$

$$\therefore \quad \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1} + 1\right]} \rightarrow (2)$$

$\gamma = \alpha + j\beta$

$|\gamma| = \sqrt{\alpha^2 + \beta^2}$

$|\gamma^2| = \alpha^2 + \beta^2$

$\gamma^2 = j\omega\mu\sigma - \omega^2\mu\varepsilon$

$|\gamma^2| = \sqrt{(\omega^2\mu\varepsilon)^2 + (\omega\mu\sigma)^2}$

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Equations for propagation constant at different media

مثالي ضياع نازل

① lossless dielectric (perfect dielectric)
(free space ε_0, μ_0)

غير موصل

$\sigma = 0$ (no conductivity)

• العوازل الفائقة هي أفضل المواد المستخدمة لنقل الموجات

$$\gamma^2 = \cancel{j\omega\mu\sigma}_{0} - \omega^2\mu\varepsilon$$

$$\gamma = \sqrt{-\omega^2\mu\varepsilon} = j\omega\sqrt{\mu\varepsilon}$$

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$$\therefore \gamma = \alpha + j\beta$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu\varepsilon} \quad \text{rad/m}$$

موصل مثالي

② perfect Conductor

$$\sigma = \infty$$

$$\therefore \alpha = \infty \qquad \text{attenuation} = \infty$$

The wave has sharp attenuation at the surface of the perfect conductor material (no field)

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عازل الفقد

② lossy dielectric

$\sigma \neq 0$ $\quad \sigma \gg \omega\varepsilon$ $\quad \left|\frac{J_c}{J_d}\right| \gg 1$

$\frac{\sigma}{\omega\varepsilon} \gg 1$ ∴

$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}$

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$ $\quad \rho = \frac{1}{\rho}$

$= \sqrt{j\omega\mu\sigma}\sqrt{1 + \frac{j\omega\varepsilon}{\sigma}}$

$= \sqrt{j\omega\mu\sigma}\,(1 + \frac{1}{2}(\frac{j\omega\varepsilon}{\sigma}) + \ldots)$

$= \sqrt{j\omega\mu\sigma}\,(1 + \frac{j\omega\varepsilon}{2\sigma})$

$= \frac{1+j}{\sqrt{2}}\sqrt{\omega\mu\sigma} + \frac{j}{2}\sqrt{\frac{\omega\mu}{\sigma}}\,\omega\varepsilon$

$\alpha = \frac{1}{2}\sqrt{\frac{\omega\mu}{\sigma}}$ $\quad Np/m$

$\beta = \omega\sqrt{\mu\varepsilon}$ $\quad rad/m$

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④ Good Conductor

$$\sigma \gg \omega\varepsilon \qquad \left|\frac{\sigma}{\omega\varepsilon}\right| \gg 1$$

$$\frac{\sigma}{\omega\varepsilon} \gg 1$$

$$\therefore \gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon} = \gamma$$

$$\gamma = \sqrt{j\omega\mu\sigma\left[1 - \frac{\omega^2\mu\varepsilon}{j\omega\mu\sigma}\right]} = \gamma$$

$$= \sqrt{j\omega\mu\sigma}\left[1 + \frac{\omega\varepsilon}{j\sigma}\right]^{1/2} =$$

$$= \sqrt{j\omega\mu\sigma}\left[1 + \frac{\omega\varepsilon}{2j\sigma}\right] \qquad \sqrt{j} = \frac{1}{\sqrt{2}}j + \frac{1}{\sqrt{2}} =$$

$$= \left(\frac{1}{\sqrt{2}}j + \frac{1}{\sqrt{2}}\right)\left(\sqrt{\omega\mu\sigma}\right)\left(1 + \frac{\omega\varepsilon}{2\sigma}\right) =$$

$$= \left(\frac{j}{\sqrt{2}}\sqrt{\omega\mu\sigma} + \frac{\sqrt{2}}{\sqrt{\omega\mu\sigma}}\right)\left(1 + \frac{\omega\varepsilon}{2\sigma}\right) =$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}} + \frac{\omega\varepsilon}{2}\sqrt{\frac{\omega\mu}{2\sigma}} + j\frac{\omega\varepsilon}{2}\sqrt{\frac{\omega\mu}{2\sigma}}$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \text{Np/m}$$

$$\beta = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \text{rad/m}$$

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Wave impedance (η)

$$\bar{E}_x = E^+ e^{-\gamma z} + E^- e^{\gamma z} \hat{a}_x$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times E = -j\omega\mu H$$

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only for forward wave (E & H)

$$\gamma E^+ e^{-\gamma z} = j\omega\mu H \rightarrow (H^+ e^{-\gamma z})$$

$$\gamma E = j\omega\mu H$$

when $R = \frac{V}{I}$

$$\therefore \eta = \frac{E}{H} = \frac{j\omega\mu}{\gamma}$$

$$\therefore \eta = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}}$$

$$= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}$$

$$= \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

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Where η is Complex

$$\eta = |\eta| e^{j\theta_\eta}$$

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$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$$

$\tan\theta_t = \tan 2\theta_\eta$ → loss tangent

$\theta = 2\theta_\eta$ → loss angle

$$\therefore E(z) = E_o^+ e^{-\gamma z} + E_o^- e^{\gamma z}$$

$$E(z,t) = Re\left(E(z)\, e^{j\omega t}\right)$$

as $E(z,t)$ is an Explicit function

$$= Re\left(E_o^+ e^{-\alpha z} e^{-j\beta z} e^{j\omega t}\right)$$

$$= E_o^+ e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_E$$

$$\therefore H = \frac{E_o^+}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)$$

$$H = \frac{E_o^-}{|\eta|} e^{\alpha z} \cos(\omega t + \beta z - \theta_\eta)$$

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Special Cases of wave impedance

① perfect dielectric

$\sigma = 0$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}$$

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② free space

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 = 120\pi \ \Omega$$

③ lossy medium (dielectric)

$$\frac{\sigma}{j\omega\varepsilon} \ll 1$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} \cdot \frac{1}{\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \left(1 + \frac{\sigma}{j\omega\varepsilon}\right)^{-1/2}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} + j\frac{\sigma}{2\omega\varepsilon}\sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} + j\frac{\sigma}{2\omega}\sqrt{\frac{\mu}{\varepsilon}}$$

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4 lossy medium (Good Conductor)

$$\frac{\omega\varepsilon}{\sigma} \ll 1$$

$$\gamma = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} \cdot \frac{1}{\sqrt{1 + j\frac{\omega\varepsilon}{\sigma}}}$$

$$= \sqrt{\frac{j\omega\mu}{\sigma}} \left(1 + j\frac{\omega\varepsilon}{\sigma}\right)^{-1/2}$$

$$= \sqrt{\frac{j\omega\mu}{\sigma}} \left(1 - \frac{j\omega\varepsilon}{2\sigma} + \cdots\right)$$

$$\sqrt{j} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$$= \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$$

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5) perfect Conductor

$$\sigma = \infty$$

γ = Zero

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Skin depth (δ)

The Signal is meant to be exist and to be detected until it reaches e^{-1} from its Max

, So e^{-1} is equally to $e^{-\alpha z}$ $(e^{-1} = 36\%)$

$\therefore -1 = -\alpha z$

$$z = \delta = \frac{1}{\alpha}$$

① perfect dielectric

$\alpha = 0$, $\delta = \infty$

The wave not suffer from any attenuation So it can travel ∞ length through the perfect dielectric (free space)

② lossy dielectric

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$$

$$\delta = \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$$

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3) Good Conductor

$$\alpha = \sqrt{\frac{\omega \mu \sigma}{2}}$$

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

② & ③ Shows that the skin depth (δ) increase as the (σ) decrease

④ perfect Conductor

$$\sigma = \alpha = \infty$$

$$\delta = \text{Zero}$$

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the Wave Cann't penetrate the perfect Conductor as it is reflected

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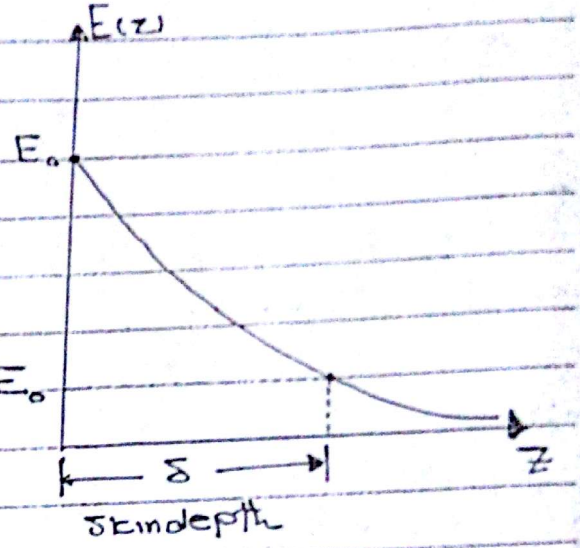
- it is clear as $\sigma \neq 0$ the wave is attenuated while it travels through the medium.



- After the signal reaches 36% from its initial value it will be very attenuated and not detectable

- The distance after which the signal attenuated is called the skin depth (δ)

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(2)

Example :

lossy dielectric has impedance $200/30°\ \Omega$

and Magnetic field $H = 10\, e^{-\alpha x} \cos(\omega t - \frac{1}{2}x)\, a_y$ A/m

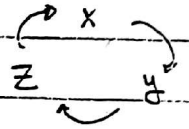
Find E and α, Skin depth ?, Wave Polarization

Sol.

The wave travels at $a_E = \hat{a}_x$

$a_H = \hat{a}_y$

$\therefore \hat{a}_E = \hat{a}_E \times \hat{a}_H$



$\therefore a_E = -\hat{a}_z$

$H = 10\, e^{-\alpha x} \cos(\omega t - \frac{1}{2}x)\, \hat{a}_y$

$H_0 = 10$

$|\eta| = \frac{E_0}{H_0}$

$\therefore E_0 = |\eta| H_0 = 10 * 200$

$\therefore E = 2000\, e^{-\alpha x} \cos(\omega t - \frac{x}{2} + \underset{\eta}{\underset{\downarrow}{\theta}} \frac{\pi}{6})\, (-a_z)$

$= -2000\, e^{-\alpha x} \cos(\omega t - \frac{x}{2} + \frac{\pi}{6})\, \hat{a}_z$ V/m

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$$\therefore \beta = \frac{1}{2}$$

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

$$\therefore \tan(2\theta_{inc}) = \frac{\sigma}{\omega\varepsilon}$$

$$= 30°$$

$$\therefore \frac{\sigma}{\omega\varepsilon} = \sqrt{3}$$

$$\therefore \frac{\alpha}{\beta} = \frac{\left(\sqrt{(1+3)}-1\right)^{1/2}}{\left(\sqrt{1+3}+1\right)^{1/2}} = \frac{1}{\sqrt{3}}$$

$$\therefore \alpha = \frac{1}{2\sqrt{3}} \text{ NP/m}$$

$$\therefore \delta = \frac{1}{\alpha} = 2\sqrt{3} \text{ m}$$

- Wave polarization → is the direction of the electric field

So, the wave is polarized along the (z) direction.

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Example :

In a lossless medium for which $\eta = 60\pi$, $\mu_r = 1$

$$H = .1 \cos(\omega t - z)\,\hat{a}_x + .5 \sin(\omega t - z)\,\hat{a}_y \text{ A/m}$$

Find E, ω, ε_r ??

→ For lossless medium

$$\sigma = 0 \qquad \alpha = 0$$

$$\because \eta = 60\pi$$

$$\therefore \eta = \sqrt{\frac{\mu}{\varepsilon}} = 60\pi$$

$$\frac{\sqrt{\mu_0 \mu_r}}{\sqrt{\varepsilon_0 \varepsilon_r}} = 60\pi$$

$$\therefore \varepsilon_r = 4$$

$$\therefore H = .1 \cos(\omega t - z)\,\hat{a}_x + .5 \sin(\omega t - z)\,\hat{a}_y$$

$$\therefore \beta = 1$$

$$\beta = \omega\sqrt{\mu\varepsilon} = 1 \qquad \therefore \omega = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\therefore \omega = 1.5 * 10^8 \text{ rad/sec}$$

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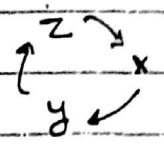
The wave propagates at z

$a_k = \hat{a}_z$

$a_H = -\hat{a}_x$ $\quad$ $a_H = \hat{a}_y$

$a_E = \hat{a}_y$ $\quad$ $a_E = \hat{a}_x$



$\therefore \eta = \frac{E_{o1}}{H_{o1}}$

$\therefore E_{o1} = \eta H_{o1}$ $\quad$ $E_{o2} = \eta H_{o2}$

$\therefore \overline{E} = 60\pi * .1 \cos(\omega t - z)\, a_y$

$+ 60\pi * .5 \sin(\omega t - z)\, a_x =$

$= 6\pi \cos(1.5 * 10^8 t - z)\, \hat{a}_y + 30\pi \sin(\omega t - z)\, \hat{a}_x$ V/m

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Example :

Auniform plane wave propagating in a medium

$$E = 2 e^{-\alpha z} \sin(10^8 t - \beta z)\, a_y \ V/m$$

If the medium is characterized by $\varepsilon_r = 1$, $\mu_r = 20$, $\sigma = 3$.

Find (α ; β ; H)

Sol.

we have to determine the type of medium → lossy dielectric / good conductor

determine the loss tanget

$$\frac{\sigma}{\omega \varepsilon} = \frac{3}{10^8 * \varepsilon_0} = 3393 \gg 1$$

The medium is a good Conductor

$$\text{ie } \alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}}$$

$$= \left(\frac{10^8 * 4\pi * 10^7 * 20 * 3}{2} \right)^{1/2}$$

$$= 61.4$$

$$\alpha = 61.4 \ Np/m$$

$$\beta = 61.4 \ rad/m$$

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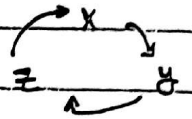
$\therefore E = 2 e^{-61.4z} \sin(10^8 t - 61.4z)\, \hat{a}_y$

$\hat{a}_k = \hat{a}_z$

$a_E = \hat{a}_y$



$\therefore a_H = -\hat{a}_x$

$H_o = \frac{E_o}{|\eta|}$

$\therefore |\eta| = \sqrt{(\text{real})^2 + (\text{imag})^2}$

$= \sqrt{\left(\sqrt{\frac{\omega\mu}{2\sigma}}\right)^2 + \left(\sqrt{\frac{\omega\mu}{2\sigma}}\right)^2} = \sqrt{\frac{\omega\mu}{\sigma}}$

$= \sqrt{\frac{800\pi}{3}}$

$\therefore \theta_{\eta} = \tan^{-1}\left(\frac{\text{img}}{\text{real}}\right) = 45^\circ = \frac{\pi}{4}$

$\& \tan(2\theta_{\eta}) = \frac{\sigma}{\omega\varepsilon}$

$\therefore \theta_{\eta} = 45^\circ = \pi/4$

$\therefore H = \frac{2}{\sqrt{\frac{800\pi}{3}}} e^{-61.4z} \sin(10^8 t - 61.4z - \pi/4)(-\hat{a}_x)$

$= -69.1 * 10^{-3} e^{-61.4z} \sin(10^8 t - 61.4z - \frac{\pi}{4})\, \hat{a}_x$

A/m

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* power and the poynting vector

$$\nabla \times E = \mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t} \quad * E$$

$$E \cdot (\nabla \times H) = \sigma E^2 + \varepsilon \frac{\partial E^2}{\partial t} \rightarrow (1)$$

$$\nabla \cdot (A \times B) = B (\nabla \times A) - A \cdot (\nabla \times B)$$
$$\downarrow H \quad \downarrow E \qquad \downarrow E \quad \downarrow H \quad \downarrow H \quad \downarrow E$$

$$\nabla \cdot (H \times E) + H (\nabla \times E) = E \cdot (\nabla \times H)$$

From (1)

$$\nabla \cdot (H \times E) + H \cdot (\nabla \times E) = \sigma E^2 + \varepsilon \frac{\partial E^2}{\partial t}$$

$$\nabla \cdot (H \times E) - \mu \frac{\partial H^2}{\partial t} = \sigma E^2 + \varepsilon \frac{\partial E^2}{\partial t}$$

$$-\nabla \cdot (E \times H) = \sigma E^2 + \frac{\varepsilon}{2} \frac{\partial E^2}{\partial t} + \frac{\mu}{2} \frac{\partial H^2}{\partial t}$$

$$\int_V \nabla \cdot (E \times H)\, dV = -\int_V \sigma E^2 \, dV - \frac{\partial}{\partial t} \int_V \left(\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2 \right) dV$$

From divergence theorem

$$\oint_S (E \times H) \cdot dS = -\int_V \sigma E^2 \, dV - \frac{\partial}{\partial t} \int_V \left(\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2 \right) dV$$

Total power

$\therefore$ poynting vector $\mathcal{P} = E \times H$ watt/m^2

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* Poynting vector ($\mathcal{P}$)

$$\mathcal{P}(z,t) = E \times H$$

* instantaneous poynting vector

$$\mathcal{P}_{ave}(z) = \frac{1}{T}\int_0^T \mathcal{P}(z,t)\,dt$$

$$= \frac{1}{2} Re\,(E \times H^*)$$

$$= \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos\theta_\eta \; \hat{a}_z$$

* The total time average power

$$P_{ave} = \int_S \mathcal{P}_{ave}(z) \cdot dS \quad \text{Watt}$$

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Example:

In a nonmagnetic medium

$E = 4 \sin(2\pi * 10^7 t - .8x)\ \hat{a}_z$ V/m

Find

① ε_r, η

② The time average power

③ the total power crossing $100\,cm^2$ of plane $2x + y = 5$

Sol.

nonmagnetic means lossless $\alpha = 0$ $\sigma = 0$ $\mu_r = 1$

① $\beta = \omega\sqrt{\mu\varepsilon} = .8$

$.8 = 2\pi * 10^7 \sqrt{\mu_0 * \mu_r \varepsilon_0 \varepsilon_r}$

$\varepsilon_r = 14.59$

$\eta = \sqrt{\frac{\mu}{\varepsilon}} = 10\pi^2 = 98.7\ \Omega$

② $\mathcal{P} = E \times H = \frac{E_0^2}{|\eta|} \sin^2(\omega t - \beta x)\ \hat{a}_x$

$\mathcal{P}_{ave} = \frac{1}{T} \int \mathcal{P}\, dt = \frac{E_0^2}{2\eta}\, a_x$

$= \frac{(4)^2}{2 * 10\pi^2} = .081\ \hat{a}_x$ W/m^2

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$$P_{ave} = \int_S \mathcal{P}_{ave} \cdot dS = \mathcal{P}_{ave} \cdot S \cdot \hat{a}_n$$

$$\therefore P_{ave} = (81 * 10^{-3} \hat{a}_x) \cdot (100 * 10^{-4}) \cdot \hat{a}_n$$

$$a_n = \frac{2\hat{a}_x + \hat{a}_y}{\sqrt{2^2 + 1^2}} = \frac{2a_x + a_y}{\sqrt{5}}$$

$$P_{ave} = .081 a_x * 100 * 10^{-4} * \frac{2a_x + a_y}{\sqrt{5}}$$

$$\hat{a}_x \cdot \hat{a}_x = 1$$

$$\hat{a}_x \cdot \hat{a}_y = \text{Zero}$$

$$P_{ave} = 8.1 * 10^{-4} * \frac{2}{\sqrt{5}}$$

$$= 724.5 \ \mu W$$

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* Reflection of A plane wave at Normal incidence

. When a wave from any medium meets a different medium ; it's partly transmitted and partly reflected due to the different medium parameters (σ , ε , μ)

. Assume a plane wave propagating along +z direction ; the wave is incident normally at $z = 0$.

. For incident wave

(E_i ; H_i) is traveling along (+ $\hat{a}_z$) ; but if reflection happens the refflected (E_r ; H_r) is travelling along ($- a_z$) .

① $E_i(z) = E_{io} \, e^{-\gamma_1 z} \, \hat{a}_x$

② $H_i(z) = \dfrac{E_{io}}{\eta_1} \, e^{-\gamma_1 z} \, \hat{a}_y$

. for reflected wave

③ $E_r(z) = E_{ro} \, e^{\gamma_1 z} \, \hat{a}_x$

④ $H_r(z) = \dfrac{E_{ro}}{\eta_1} \, e^{\gamma_1 z} \, (-\hat{a}_y)$

لاحظ أن الـ wave بتكون على شكل power

① الـ power بتغير إتجاهها
② شدة الـ (E) اتجاهها ،،
③ بينما الـ (H) اتجاهها متغير ،، والعكس صحيح

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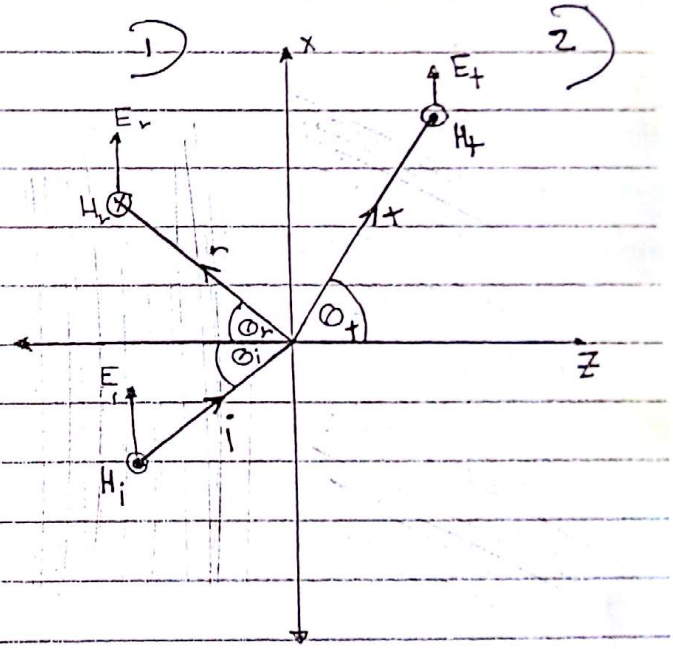
. For Transmitted wave

⑤ $E_t(z) = E_{to}\, e^{-\gamma_2 z}\, \hat{a}_x$

⑥ $H_t(z) = \frac{E_{to}}{\eta_2}\, e^{-\gamma_2 z}\, \hat{a}_y$

(E_t, H_t) is traveling along $(+a_z)$



. at medium ①

$\sigma_1, \mu_1, \varepsilon_1$

$E_1 = E_i + E_r$

↳ electric field at medium ①

$H_1 = H_i + H_r$

↳ Magnetic field at medium ①

. at medium ②

$\sigma_2, \mu_2, \varepsilon_2$

$E_2 = E_t$

$H_2 = H_t$

* Γ → reflection co.eff.
* τ → transmission co.eff.
* $1 + \Gamma = \tau$
* Γ and τ are dimensionless
* $0 \leq |\Gamma| \leq 1$

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* at the interface ($z = 0$) ; applying the boundary conditions (tangential component of E & H must be continous).

means

$$E_{1\tan} = E_{2\tan} \quad ; \quad H_{1\tan} = H_{2\tan}$$

$$E_{1\tan} = E_i(0) + E_r(0)$$

$$E_{2\tan} = E_t(0)$$

$$H_{1\tan} = H_i(0) + H_r(0)$$

$$H_{2\tan} = H_t(0)$$

$$\therefore E_i(0) + E_r(0) = E_t(0)$$

$$E_{io} + E_{ro} = E_{to} \longrightarrow (1)$$

also

$$H_{io} + H_{ro} = H_{to}$$

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2} \longrightarrow (2)$$

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From ① & ②

$$\therefore \Gamma = \frac{E_{ro}}{E_{io}}$$

$$\therefore \frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{io} + E_{ro}}{\eta_2}$$

$$E_{io}\left(\frac{1}{\eta_1} - \frac{1}{\eta_2}\right) = E_{ro}\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right)$$

$$E_{io}\left(\frac{\eta_2 - \eta_1}{\eta_2 \cdot \eta_1}\right) = E_{ro}\left(\frac{\eta_2 + \eta_1}{\eta_2 \cdot \eta_1}\right)$$

$$\therefore \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma$$

and $\tau = \frac{E_{to}}{E_{io}}$

$$\therefore \frac{E_{io}}{\eta_1} - \frac{E_{to} - E_{io}}{\eta_1} = \frac{E_{to}}{\eta_2}$$

$$E_{io}\left(\frac{2}{\eta_1}\right) = E_{to}\left(\frac{1}{\eta_2} + \frac{1}{\eta_1}\right)$$

$$E_{io}\left(\frac{2}{\eta_1}\right) = E_{to}\left(\frac{\eta_2 + \eta_1}{\eta_2 \cdot \eta_1}\right)$$

$$\frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1} = \tau$$

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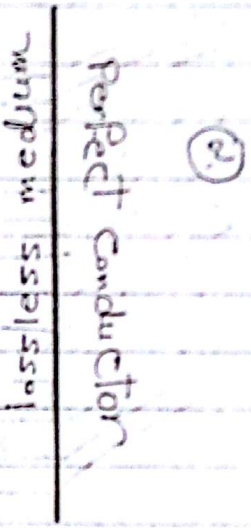
* $1 + \Gamma = \tau$ where $0 \leq |\Gamma| \leq 1$

→ Special Case

- medium ① is
  (perfect dielectric
  lossless medium
  free space)

$\sigma_1 = 0$ $\alpha_1 = 0$

- medium ② is
  (perfect conductor)



at this case ; All incident wave will be reflected to the medium ① .

(E_i & E_r) has equal amplitude but in opposite direction .

∴ $\Gamma = -1$ $\sigma_1 = 0$

$\alpha_1 = 0$

$\gamma_1 = j\beta_1$

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So, E_1 will be standing wave

$$\therefore E_{1S} = E_{iS} + E_{rS}$$

$$= E_{io} e^{-\gamma_1 z} + E_{ro} e^{\gamma_1 z}$$

and $E_{io} = -E_{ro}$

$$\therefore E_{1S} = E_{io} \left(e^{-j\beta_1 z} - e^{j\beta_1 z} \right)$$

$$= -2j E_{io} \sin \beta_1 z$$

$$E_{(z,t)} = -2j E_{io} \sin(\beta_1 z)\, e^{j\omega t}$$

$$= -2j E_{io} \sin(\beta_1 z) \left[\cos \omega t + j \sin \omega t \right]$$

$$E_{1(z,t)} = \mathrm{Re}\left(E_{1S} e^{j\omega t} \right)$$

$$= 2 E_{io} \sin(\beta_1 z) \sin(\omega t)\, \hat{a}_x$$

and $H_1 = \frac{2 E_{io}}{\eta_1} \cos(\beta_1 z) \cos(\omega t)\, \hat{a}_y$

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So ; E_1 will be standing wave

$$\therefore E_{1S} = E_{iS} + E_{rS}$$

$$= E_{i0} e^{-\gamma_1 z} + E_{r0} e^{\gamma_1 z}$$

and $E_{i0} = -E_{r0}$

$$\therefore E_{1S} = E_{i0} \left(e^{-j\beta_1 z} - e^{+j\beta_1 z_1} \right)$$

$$= -2j \sin \beta_1 z$$

$$= -2j E_{i0} \sin \beta_1 z$$

Standing wave ratio

is the ratio between $|E_1|_{max}$ and $|E_1|_{min}$

or the ratio between $|H_1|_{max}$ and $|H_1|_{min}$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

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Examples

o The electric field is given by

$$E = 5_o \cos(10^8 t + \beta x)\, \hat{a}_y$$

Find

a) The direction of wave propagation

b) The type of medium

c) β and time it takes to travel a distance of $\lambda/2$

d) sketch the wave at $t = 0$; $T/4$; $T/2$

Solution :

(a) From positive sign $(\omega t + \beta x)$

So, the wave propagates at $-\hat{a}_x$

(b) $\because \gamma = j\beta$ ∴ this medium is lossless medium (free space)

Because (ε_r & μ_r) not given

assumed equal = (1)

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(c)

In free space $u = c$

$$\beta = \frac{w}{c} = \frac{10^8}{3*10^8} = \frac{1}{3} \text{ rad/m}$$

if T is the time period so, λ will be the distance it takes at (T) when travel with c velocity

$$\text{or } V = \frac{x}{t} \qquad \text{or } \lambda = c \cdot T$$

$$\text{where } \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{1/3} = 6\pi \text{ m}$$

$$\text{or } T = \frac{\lambda}{c} = 62.8 \text{ ns}$$

and $(\lambda/2)$ has $(T/2)$ time

$$t = \frac{T}{2} = 31.4 \text{ ns}$$

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a)

$t = 0$

$$E = 5_0 \cos(Bx)\ \hat{a}_y$$

$t = \frac{T}{4}$

$$E = 5_0 \cos\left(Bx + \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\right)$$

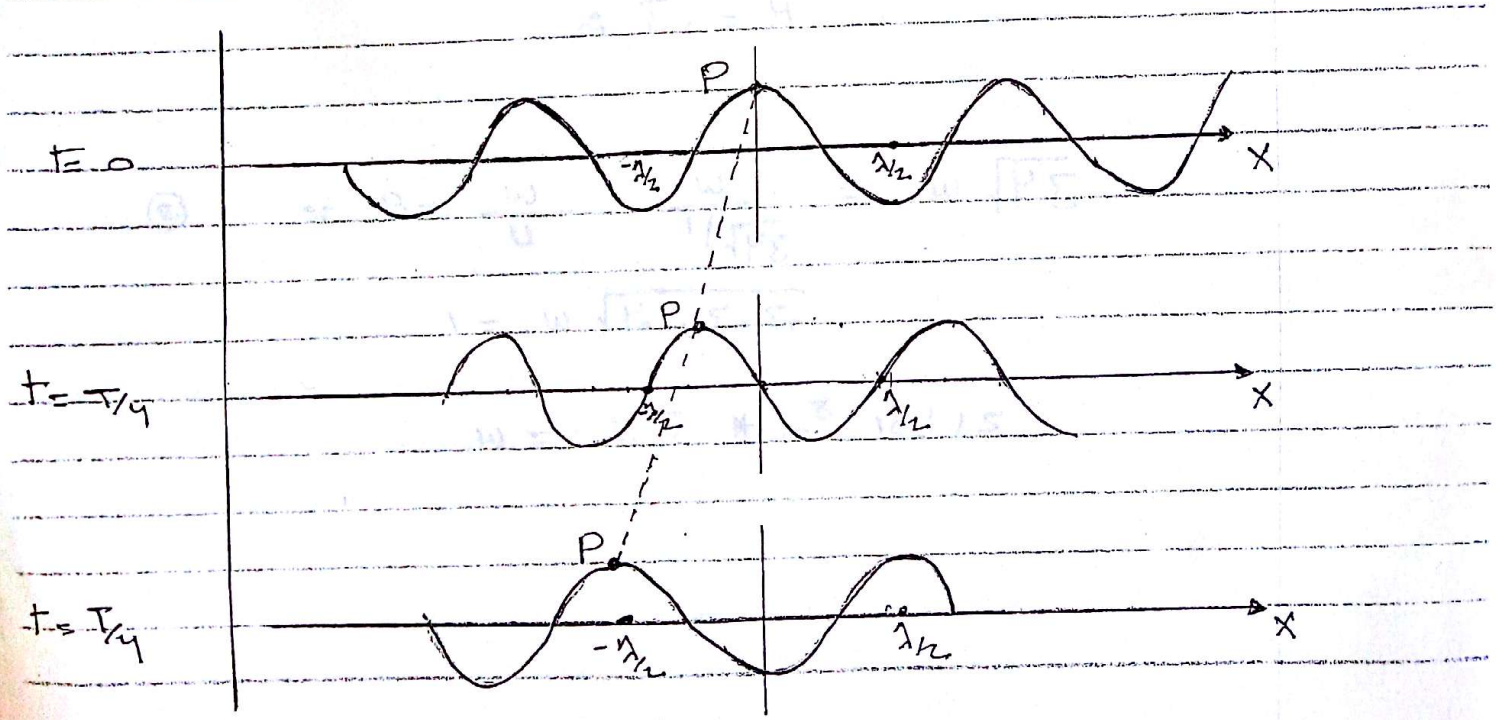
$$= 5_0 \cos\left(Bx + \frac{\pi}{2}\right)\ \hat{a}_y$$

$$= -5_0 \sin(Bx)\ \hat{a}_y$$

at $t = \frac{T}{2}$

$$E = 5_0 \cos(Bx + \pi)$$

$$= -5_0 \cos(Bx)\ \hat{a}_y$$



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②

a medium has $\eta = 60\pi$; $\mu_r = 1$ and $H = -.1 \cos(\omega t - z) a_x + .5 \sin(\omega t - z) a_y$ A/m

- Calculate ε_r ; ω ; E

Solution :

First determine the type of the medium

$(\omega t - \beta z)$ $\alpha = dB$ is lossless medium

$\therefore \sigma = 0$ $\alpha = 0$ $\beta = 1$

① $\therefore \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = 60\pi$

$\therefore \varepsilon_r = 4$

② $\therefore \beta = \frac{\omega}{u} = \frac{\omega}{1/\sqrt{\mu\varepsilon}} = \omega\sqrt{\mu\varepsilon}$

$1 = \omega \sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}$

$\omega = 1.5 * 10^8$ rad/s

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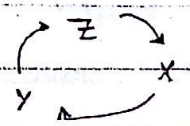
③

$H = -.1 \cos(\omega t - z)\, \hat{a}_x + .5 \sin(\omega t - z)\, \hat{a}_y$ A/m

$\therefore H = H_x + H_y$

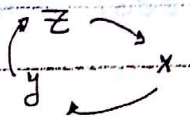
and the wave propagates at $+\hat{a}_z$ Direction



$\square \times a_x = a_z$

$-a_y \times a_x = a_z$

$E \longrightarrow -a_y$



$\square \times a_y = a_z$

$a_x \times a_y = a_z$

$E = a_x$

where $E = \eta H$

$\therefore E = 60\pi * -.1 \cos(\omega t - z)\,(-a_y)$

$+ 60\pi\,(.5) \sin(\omega t - z)\, a_x$

$= -30\pi \sin(\omega t - z)\, a_x + 60\pi \cos(\omega t - z)\, \hat{a}_y$

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Q

In free space ($z \geq 0$), a plane wave with

$$H = 1_0 \cos(10^8 t - \beta z)\, a_x \quad mA/m$$

Determine (E_r, H_r), (E_t, H_t)

if the wave is incident normally on a lossless medium ($\varepsilon = 2\varepsilon_0$, $\mu = 8\mu_0$)

Solution:

Free space | lossless dielectric

$$\beta_1 = \frac{\omega}{c} = \frac{10^8}{3\times10^8} = \frac{1}{3}$$

$$\eta_1 = \eta_0 = 120\pi$$

$$\beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = 4\frac{\omega}{c} = \frac{4}{3}$$

$$\eta_2 = \sqrt{\frac{\mu_r}{\varepsilon_r}}\,\eta_1 = 2\eta_1$$

$$H_i = 1_0 \cos(10^8 t - \tfrac{1}{3}z)\, \hat{a}_x$$

$$E_i = -1_0 * 120\pi \cos(10^8 t - \tfrac{1}{3}z)\, \hat{a}_y \quad mV/m$$

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$$\therefore \Gamma = \frac{E_{ro}}{E_{io}} = \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} = \frac{1}{3}$$

$$\therefore E_{ro} = \frac{1}{3} E_{io}$$

$$\therefore E_r = \frac{-10 * 120\pi}{3} \cos(10^8 t + \tfrac{1}{3} z)\, \hat{a}_y \quad mV/m$$

$$\therefore H_r = \frac{-10}{3} \cos(10^8 t + \tfrac{1}{3} z)\, \hat{a}_x \quad mA/m$$

$$\therefore 1 + \Gamma = \tau \qquad \tau = \tfrac{4}{3}$$

$$\therefore E_t = \frac{40}{3} * 120\pi \cos(10^8 t - \tfrac{4}{3} z)\, \hat{a}_y \quad mV/m$$

$$H_t = \frac{E_t}{\mu_2} = \frac{E_t}{2\mu_0}$$

$$H_t = \frac{20}{3} \cos(\omega t - \tfrac{4}{3} z)\, \hat{a}_x \quad mA/m.$$

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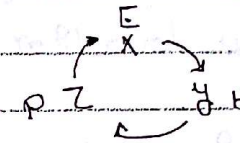
④

A 5 GHz uniform plane wave $E_{is} = 1_o e^{-j\beta z} \hat{a}_x$ in free space is incident normally on a large plane lossless dielectric ($\varepsilon = 4\varepsilon_o$; $\mu = \mu_o$)

Solution :

$E_{i-s} = 1_o e^{-j\beta_1 z} \hat{a}_x$



$H_{i-s} = \frac{1_o}{\eta_1} e^{-j\beta_1 z} a_y$

$= \frac{1_o}{120\pi} e^{-j\beta_1 z} \hat{a}_y$

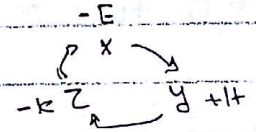
$\beta_1 = \frac{w}{c} = \frac{5*10^9}{3*10^8}$

$\beta_2 = 5*10^9 \sqrt{\mu_2 \varepsilon_2}$

$\eta_2 = \sqrt{\frac{\mu_o \mu_r}{\varepsilon_o \varepsilon_r}} = \frac{\eta_o}{2}$

$\therefore \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{-1}{3}$ $\quad \therefore \tau = \frac{2}{3}$

$\therefore E_{rs} = \frac{-1_o}{3} e^{+j\beta_1 z} \hat{a}_x$



$H_{rs} = \frac{1_o}{3*120\pi} e^{+j\beta_1 z} \hat{a}_y$

$\therefore E_{ts} = \frac{2_o}{3} e^{-j\beta_2 z} \hat{a}_x$

$H_{ts} = \frac{2_o}{3*60\pi} e^{-j\beta_2 z} \hat{a}_y$

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given a uniform plane wave in air as

$$E_i = 4_0 \cos(\omega t - \beta z)\,\hat{a}_x + 3_0 \sin(\omega t - \beta z)\,\hat{a}_y$$

(a) Find H_i

b) if the wave encounts a perfectly conductor plate normal to the z axis at $z = 0$. Find the reflected wave

c) what are the total E and H

d) Calculate the time average poynting vector for $z \geq 0$ & $z \leq 0$

Solution s

$$\eta_1 = \eta_0 \qquad \beta_1 = \frac{\omega}{c}$$

(a) $$H_i = \frac{E_i}{\eta_1} = \frac{4_0}{120\pi}\cos(\omega t - \beta_1 z)\,\hat{a}_y - \frac{3_0}{120\pi}\sin(\omega t - \beta_i z)\,\hat{a}_x$$

(b) $\Gamma = -1 \qquad \tau = 0 \qquad \sigma = \infty$

$$E_r = -4_0 \cos(\omega t + \beta_1 z)\,\hat{a}_x - 3_0 \sin(\omega t + \beta_1 z)\,\hat{a}_y$$

$$H_r = -\frac{4_0}{120\pi}\cos(\omega t + \beta_1 z)\,\hat{a}_y - \frac{3_0}{120\pi}\sin(\omega t + \beta_1 z)\,\hat{a}_x$$

(c)

$$E_{tot} = E_i + E_r \qquad H_{tot} = H_i + H_r$$

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d)

$$\overline{P} = \overline{E} \times \overline{H}$$

$$P_{ave} = \frac{|E|^2}{2\eta} \hat{a}_k$$

at $z \leq 0$

$$P_{ave} = \frac{|E_1|^2}{2\eta_1} \hat{a}_z$$

$$= \frac{|E_{io}^2 - E_{ro}^2|}{2\eta_1} \hat{a}_z$$

$$= Zero$$

at $z \geq 0$

$$P_{ave} = \frac{|E_2|^2}{2\eta_2}$$

$$= \frac{|E_t|^2}{2} \hat{a}_z = Zero$$

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④

A plane wave with $E = 50 \sin(\omega t - 5x)\ \hat{a}_y$ V/m

in a lossless medium encounters a lossy medium

($\mu_1 = 4\mu_0$, $\varepsilon_1 = \varepsilon_0$) ($\mu_2 = \mu_0$; $\varepsilon_2 = 4\varepsilon_0$)

$\sigma = .1$ mhos/m

Find

a) Γ ; τ ; S

b) E_r ; H_r

c) E_t ; H_t

d) The time average poyinting vector in the both region ← self

Solution :

	lossless	lossy
$\sigma_1 = 0$	$\alpha_1 = 0$	$\beta_1 = 5$
$\beta_1 = \frac{\omega}{u}$	$\omega = 7.5 * 10^8$	

(a) $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{4\mu_0}{\varepsilon_0}} = 2\eta_0$

$\eta_2 = |\eta_2| \, e^{j\theta_{\eta_2}}$

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$$|\eta_2| = \frac{\sqrt{\mu_2/\varepsilon_2}}{\left[1+\left(\frac{\sigma_2}{\omega\varepsilon_2}\right)^2\right]^{1/4}}$$

$$= \frac{\sqrt{\mu_0/4\varepsilon_0}}{\left[1+\left(\frac{\cdot 1}{7.5\times 10^3 \times 4\varepsilon_0}\right)^2\right]^{1/4}}$$

$$= 95.445$$

$$\tan 2\theta_{\eta_2} = \frac{\sigma}{\omega\varepsilon}$$

$$\theta_{\eta_2} = 37.57^\circ$$

$$\eta_2 = 95.445 \angle 37.57^\circ$$

$$\therefore \Gamma = \frac{95.445\angle 37.57 - 240\pi}{95.445\angle 37.57 + 240\pi} = \frac{680.8\angle 175.09}{831.67\angle 4.02}$$

$$= .818\angle 171.08^\circ$$

$$\therefore \tau = 1+\Gamma = .229\angle 33.5$$

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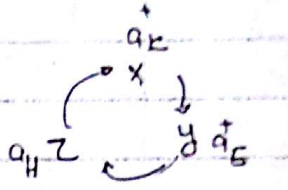
$$S = \frac{1+|\Gamma|}{1-|\Gamma|} \cong 10$$

(b) $E_i = 50 \sin(\omega t - 5x)\ \hat{a}_y$ V/m



$$H_i = \frac{50}{2\eta_0} \sin(\omega t - 5x)\ \hat{a}_z \text{ A/m}$$

$$\therefore E_r = 50 * .818 \sin(\omega t + 5x + 171.08)\ \hat{a}_y$$

$$H_r = \frac{50 * .818}{2\eta_0} \sin(\omega t + 5x + 171.08)\ \hat{a}_z$$

(c)

$$E_t = 50 * .229 \sin(\omega t - \beta_2 x + 33.5)\ \hat{a}_y \text{ V/m}$$

$$H_t = \frac{50 * .229}{2\eta_0} \sin(\omega t - \beta_2 x + 33.5)\ \hat{a}_z \text{ A/m}$$

$$\beta_2 = \frac{\omega}{u_2} = \frac{7.5 * 10^8}{1/\sqrt{\mu_0 \mu_r \varepsilon \varepsilon_r}}$$

$$= 7.826$$

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